2019 IEEE Intelligent Transportation Systems Conference (ITSC)
Auckland, NZ, October 27-30, 2019

Introducing the Effect of Bicyclist Stabilization Control in Microscopic Traffic Simulation

Georgios Grigoropoulos, Heather Kaths, and Fritz Busch

Abstract— **A bicycle dynamics model is developed that introduces characteristics of the bicycle dynamics in microscopic traffic simulation. The model alters the lateral position of simulated bicycles in space due to the stabilization efforts of the simulated bicyclist. The development of the proposed model is based on conclusions and simplifications derived from the study of single-track vehicle dynamic models. The proposed model is subsequently integrated with the microscopic traffic simulation software SUMO [1]. Results indicate that the operational width of a bicyclist is a decreasing function of the operational speed as a result of the bicyclist stabilization process. The proposed model can be used to improve the accuracy of simulation studies of the overtaking behavior of bicyclists and while studies focusing on the behavior of bicycles in platoons are expected to produce more accurate results.**

I. INTRODUCTION

Microscopic traffic simulation tools are used to model the behavior of road users and provide means of assessment for traffic measures in road networks. As the number of bicycles increases, the capacity and efficiency of the road network are significantly affected by bicycle traffic, making it necessity to accurately model of bicycle traffic. Currently, the modelling of bicycles for microscopic traffic simulation tools is achieved using modified motor vehicle or pedestrian models [1], [2]. In this context, special characteristics of bicyclist behavior such as switching between different infrastructures, flexible maneuvering between other traffic participants and desired path at intersections in complex interactions with other road users are not included in the traffic simulation or cannot be easily reproduced in the simulation, due to the inherently complex bicyclist behavior as well as the design structure of the available microscopic traffic simulation software. As a result, additional effort is required in order to incorporate special attributes of bicyclist behavior into microscopic traffic simulation.

Most of the extensions developed for bicycle simulation focus primarily on the description of the tactical behavior of bicyclists [3]–[6] as well as the modelling of the operational characteristics of bicyclist behavior [7]–[10]. The tactical behavior includes short term maneuvers as consciously chosen by a bicyclist to address specific traffic situations. The operational behavior includes subconscious actions to control and ride the bicycle in the environment, such as desired speed control, acceleration, deceleration, gap acceptance and spacing [2], [11]. Although several models that vary in complexity and accuracy have been developed in the past to describe bicycle dynamics and physics, little knowledge derived from these studies has been introduced into microscopic traffic simulation. This is attributed mainly to the fact that most of these models are highly complex and require complex calculations, which will lead to an unnecessary degree of detail and a significant decrease in simulation time, making them inappropriate for a direct implementation in microscopic traffic simulation. In this paper, a model that introduces characteristics of the bicycle dynamics into microscopic traffic simulations proposed. The model alters the lateral position of simulated bicycles in space due to the stabilization control of the bicyclist. The development of the model is based on conclusions and simplifications derived from the study of linear bicycle models.

II. LITERATURE REVIEW

A. Vehicle Dynamics in Microscopic Traffic Modelling

Vehicle dynamics have already been introduced in some microscopic traffic simulation studies. The main purpose behind these implementations was to ensure the proper operation of the Social Force Model by translating the applied social forces into realistic travel direction changes. In the case of car-following models primarily utilized by microscopic traffic simulation software the agent's behavior is altered based on the position and speed of the leading vehicle. The lateral movement is modelled through a discrete lane choice model [2] and in the case of mixed traffic simulation through the resolution of a traffic lane into sub-lanes [12] or through non-lane-based behavior using the maximum longitudinal time-to-collision to choose the lateral position along a traffic lane [13], [14]. Mixed traffic in shared space is modelled with an adapted Social Force Model in [15]. A dynamic model is integrated for four-wheel motor vehicles that can accurately simulate the vehicle trajectory in space, considering the restrictions imposed by its geometry and physics. The sum of all social forces acting on the vehicle are transformed into a steering angle using a Proportional-Integral-Derivative (PID) controller. In this case, the lateral component of social force works as the "error". The output of the PID controller is the control input (steering angle) for the controlled system (vehicle). Thus, the target of the PID controller is to determine the steering angle while minimizing the error of the social force. This approach

---

Georgios Grigoropoulos is with the Chair of Traffic Engineering and Control, Technical University of Munich (TUM), Munich, Germany (phone: +49 (89) 289 - 28584; e-mail: georgios.grigoropoulos@tum.de).

Heather Kaths is with the Chair of Traffic Engineering and Control, Technical University of Munich (TUM), Munich, Germany (e-mail: heather.kaths@tum.de)

Fritz Busch is with the Chair of Traffic Engineering and Control, Technical University of Munich (TUM), Munich, Germany (e-mail: Fritz.Busch@vt.bv.tum.de).

978-1-5386-7024-8/19/$31.00 ©2019 IEEE

can be adapted for use with other types of vehicles that are to be simulated with the Social Force Model.

Vehicle dynamics were considered in the development of a microscopic traffic simulation model in [4]. The Social Force Model was used as a basis for modelling mixed traffic in shared space areas. Extensions are introduced to the Social Force Model, in order to represent the interactions among the different types of road users with a greater accuracy. An infrastructure model was also introduced that used force vector fields to guide each agent to their destination based on the type of infrastructure.

Additionally, an operational model was created for the different road users so that the different characteristics of pedestrian and vehicle dynamics could also be taken into consideration in the calculation process of the vehicle trajectory. In the case of vehicles, however, no distinction between motorized vehicles and bicycles was made. The operational model also handled basic social interactions among the agents. Finally, a tactical model was also introduced for handling conflicts among different agents that cannot be resolved by the Social Force Model [4].

B. *Bicycle Dynamics*

The study of bicycle dynamics bicycle self-stabilization and human bicycle control has intrigued scientists since the invention of the bicycle. Since the 19th century, a series of linear and non-linear models have been developed to describe bicycle dynamics [16]–[20]. Although most of these models are complex and importing them into microscopic traffic simulation in order to describe the bicycle dynamics accurately is deemed as an unnecessary step towards improving the accuracy of microscopic traffic simulation, certain characteristics of the bicycle dynamics can be modelled in a simplified way and improve the accuracy without reducing the simulation efficiency. A similar scientific approach is followed in [16] where four models with a gradually increasing accuracy and complexity are presented, with the last model being a nonlinear model based on fourth order differential equations. The simplest model presented is a second order bicycle model that relies on several simplifying assumptions. The single control input to the system is the steering torque, while the bicycle is described by the dynamic torque balance for the frame and a static momentum balance for the front fork assembly. Due to the simplifications related to the bicycle geometry that reduce angular parameters to zero, the equations become linearized. A closed loop control system then describes the stabilization of the bicycle, where steering torque is the only input to the system. Although the model is simple, solving the equations requires a laplacian transformation, reducing the applicability of the model in microscopic traffic simulation.

More complex models also discussed in [16], include a linear fourth order model, where the static front fork is modelled with a dynamic model. The model is defined by 23 parameters, which describe the bicycle geometry and mass distribution. Linearized equations for a fourth order bicycle model based on the Whipple Bicycle Model [21] have also been developed by [20] and can be used for studying bicycle self-stabilization. Different modes of bicycle stability can be represented by the models. A significant conclusion that can be derived is that an uncontrolled bicycle is only self-stable at speeds between 15.5 km/h (4.3 m/s) and 21.6 km/h (6 m/s) [20]. A similar approach, based on the Whipple Bicycle Model, in the study of bicycle self-stability but without linearized equations was already proposed in [17], which produced similar results. It was determined that an uncontrolled bicycle can be self-stable at speeds between 15.9 km/h (4.4 m/s) and 20.2 km/h (5.6 m/s). Finally, both models show that the change of roll and steering angle can be described by exponential functions when it comes to controlling the stability of the bicycle.

When stabilizing a bicycle without changing the direction of travel, bicyclists primarily rely on steering [19], [22]–[25] The motion of the bicyclists body while performing stability tasks on a treadmill was captured using a motion capture system. It is determined that steering-yaw-roll motions of the bicyclist were taking place with a greater frequency for speeds below 10 km/h, while for speed over 10 km/h, the predominant motion of the bicyclist was the pedaling motion [19]. The frequency of the steering-yaw-roll motions matched the frequency of the pedaling motion of the bicyclist. Researchers conducted bicycle experiments on a treadmill together with a town ride experiment. Observation data showed small steering angles of ±3 degrees for most of the field study. Steering angles of ±15 degrees were recorded only when the bicyclist was moving at low speed or turning. The laboratory bicycle experiment that was conducted in parallel showed that lower cruising speeds lead to greater steering angle deviations. It also suggested that steering angle deviations are continuously present during the stabilization process [26].

Finally, it is found that the change in the amplitude of the steering angle is a function of different speeds and different pedaling profiles. As the speed decreases, the amplitude of the steer angle also decreases. The pedaling profile of the bicyclist also affects the amplitude of the steer angle variability. When the bicyclist is not pedaling, the amplitude of the steer angle variation is smaller in comparison to normal pedaling [27]. Thus, it can be concluded that the stabilization process of the bicycle also affects the position of the bicycle in space, because when moving forward, the bicyclist stabilizes the bicycle through changes in the steering angle. The frequency of these changes in the steering angle are linked to the pedaling frequency, whereas the amplitude of the steering angle changes is linked to the pedaling profile of the bicyclist as well as the bicycle speed. Also, the amplitude of the changes in the steering angle follows an exponentially decreasing function of velocity. As the intensity and the frequency of these changes depend on continuously changing environmental factors such as the wind, the weather conditions, the slope and the pavement type [28], the lateral position of the bicycle changes over time leading to an increased and variable operational width for bicycles in space as they cannot follow a straight line due to the stabilization process [26]. This is also evident in the recommended minimum width for bicycle lanes as one study suggests [29].

C. *Review Summary*

Existing bicycle dynamics models are too complex for an efficient integration in microscopic traffic simulation.

Vehicle dynamics have been successfully introduced in microscopic traffic simulations mainly as a part of social force models. Improved accuracy of the introduction of vehicle dynamics can be achieved by considering infrastructure and road user type in resolving interactions and calculating the trajectory of the simulated users.

Concerning the stabilization control of the bicycle, bicyclists primarily rely on steering. Finally, the change in the amplitude of the steering angle is a function of speeds and pedaling profile, with an increasing bicycle speed decreasing the steering angle deviation exponentially. This is also affected by environmental factors.

III. METHODOLOGY

Based on the findings that derive from the scientific literature review, a model that introduces the effects of bicycle dynamics in microscopic traffic simulation is developed. The model must retain a low degree of complexity, as the microscopic traffic simulation analysis must retain its efficiency. Additionally, the model must be able to function within the microscopic traffic simulation without conflicting with any of the simulation functions. This is important because the model alters the position of the bicycles in every simulation step, which potentially leads to conflicts with the simulation’s tactical model and other simulated road users. Finally, the developed model will be integrated in microscopic traffic simulation to evaluate its operation.

The proposed model reproduces the effect of the stabilization control of the bicyclist in microscopic traffic simulation. As determined in the literature review, bicyclists stabilize the bicycle through steering control, which results in changes in the position of the bicycle. Thus, the bicycle cannot follow a straight path but rather a waveform trajectory. The effect of these path deviations can potentially affect overtaking maneuvers of other bicyclists especially in cases of little available space. Therefore, the model must retain certain key qualities. Regarding the stabilization process, the bicyclist makes changes in the steering angle in order to stabilize the bicycle while the bicyclist’s body does not take part in the stabilization process. Also, due to the caster angle, the angular displacement of the bicycle steering axis from the vertical axis of the front bicycle wheel, the change of the steering angle is not equal to the resulting effective steering angle. Thus, in contrast to a simple sinusoidal model for the generation of wave form like trajectories for simulated bicycles, the proposed model accounts also for the bicycle geometry and has also the potential to include bicycles with irregular frame geometry such as recumbent bicycles, prone bicycles and chopper bicycles. Additionally, the generated trajectories do not follow predictable patterns as the amplitude and the phase would dictate for a sinusoidal function.

The mathematical expressions for calculating the resulting steer angle deviation for every simulation step are developed based on a combination of the expressions describing the stabilization process of uncontrolled bicycles [17], [20] and on the experimental results found in the scientific literature [26], [27]. The deviation of the steering angle can be represented as an exponentially decreasing function of the velocity. Based on this simplification, a simplified exponential function that determines the maximum allowed deviation of the steering angle as a result of the stabilization process in every simulation step is developed. The generic form of the equation that describes the change in the steering angle amplitude as a function of the velocity in every simulation step is:

$$\delta(s) = RAe^{\lambda v(s)} + b \tag{1}$$

Where:

$\delta(s)$	Steering angle at simulation step s [º]
R	Randomization parameter [-]
$v(s)$	Bicycle velocity at simulation step s [m/s]
A, λ, b	Calibrated parameters [-]

The changes in the steering angle are introduced by the bicyclist, not only for stabilizing the bicycle but also for countering the effects of environmental factors that cannot be modelled and quantified in detail. The calibration of the function is based on average values resulting from [26], [27]. In conjunction with the findings in [26], [27], two dynamic states are defined for the simulated bicyclist: “pedaling“ for the acceleration state and “not pedalling“ for the deceleration state in order to account for the higher steering angle deviations observed when a bicyclist is pedaling and the smaller changes in the steering angle deviations when the bicyclist is not pedaling.

Environmental factors that may affect the bicyclists’ trajectory are accounted using a randomization parameter R. The main function of this parameter is to provide a range of different steering angle deviation values in every simulation step, which reproduce the effect of unquantified environmental factors on the stabilization process. The maximum limit of the randomization value cannot be strictly defined as it refers to the maximum value the steering angle deviation can receive for a speed v and is related to the environmental factors that affect bicycle motion. Fig. 1 presents the steering angle deviation for different progression speeds and without the influence of environmental factors (R = 1).

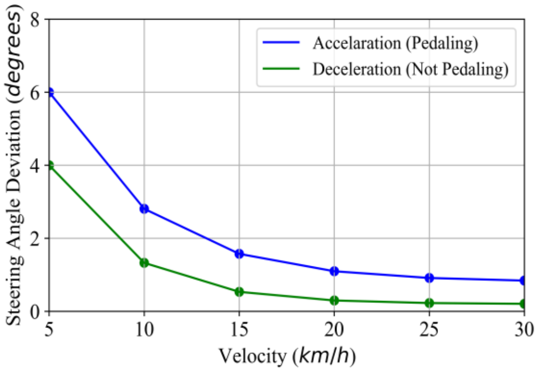

Figure 1. Steering angle deviation at different pedaling profiles with increasing speed without the influence of environmental factors (R = 1).

The second equation is introduced determine the effective steering angle for the simulated bicycle. The bicyclist does not lean in order to balance the bicycle while riding straight (roll angle $\varphi = 0^{\circ}$). The effective steering angle is the angle the direction of the bicycle changes, as a result of the caster angle and is calculated using the following equation for a simple two-wheel vehicle equation [30]:

$$\Delta(s) = \tan^{-1}\ (\cos \varepsilon \tan \delta(s)) \qquad (2)$$

Where:

$\Delta(s)$	Effective steering angle at simulation step s [º]
ε	Caster angle ($0^{\circ} \leq \varepsilon \leq 28^{\circ}$) [20] [º]
$\delta(s)$	Steer angle δ at simulation step s [º]

Finally, in order to retain the simplicity of the model and incorporate the model into microscopic simulation, a series of constraints are introduced. The frequency of the changes in the steering angle are found to be linked to the pedaling frequency. The pedaling frequency is not only a subject of the present velocity but it also depends on the geometric characteristics of every bicycle and the selected gear. As a result, an inclusion of the pedaling frequency in the proposed model is not possible. Additionally, the increase in complexity would not significantly extend the model's accuracy. The first constraint limits the number of possible changes in the travel direction of the bicycle in every simulation second as a result of the stabilization process [26].

$$c_{max} \geq \sum_{\iota=1}^{n} c_i \qquad (3)$$

Where:

c_{max}	Maximum allowed number of stabilization counteractions that are performed by the bicyclist in one simulation second [-]
c_i	Performed number of counteractions in simulation step i [-]
n	Resolution of one simulation second [s]
i	Simulation step

The second constraint limits the lateral space inside of which the changes in the lateral position of the bicycle can take place and confines the simulated bicyclist in the chosen road infrastructure. Since the magnitude of the steering angle deviation is not the same for every simulation step due to the randomization factor, the simulated bicycle is expected to diverse significantly from the desired course in the long term. As the model introduces the dynamics of a controlled bicycle in microscopic traffic simulation, a function that incorporates the bicycle dynamics with the bicyclist destination and route choice should be introduced. Ultimately the steering actions of the bicyclist do not only account for the stabilization of the bicycle but also preserve a specific path to a desired destination. Thus, the changes in the lateral position of the bicycle must be confined.

$$\mathrm{latd}_{max} \geq \sum_{\iota=1}^{n} dy_i \qquad (4)$$

Where:

latd_{max}	Maximum aggregated lateral deviation away from the desired path (m)
dy_i	Lateral deviation in simulation step i (m)
n	Resolution of one simulation second
i	Simulation step

Whenever one of the first two constraints is reached, the travel direction of the bicycle is changed. The combination of the previous two constraints produces bicycle trajectories with wavelengths and amplitudes that vary in every simulation second.

The third constraint is complementary to the first and second constraints and supports the tactical behavior of the bicyclist. It comprises a set of rules. The previous two constraints function properly only when considering bicycles advancing freely in space. However, bicycle motion is framed by the dimensions of the present bicycle infrastructure. Also, during overtaking maneuvers, the bicyclist will shift the bicycle away from the simulated road user that bicyclist is interacting with. Therefore, the third constraint is introduced in the model so that the simulated bicycle does not change infrastructure only due to the lateral deviations. In the case of overtaking maneuvers, changes in the steering angle due to the stabilization actions of the bicyclist are restricted if these actions hinder the overtaking maneuver. The methodology for applying this set of rules is linked to the restrictions and the structure of the traffic simulation software and the way interactions with agents whose behavior is altered by external models and agents simulated solely by the software are handled.

IV. RESULTS

The proposed model is integrated with the microscopic traffic simulation software SUMO [1] for the purpose of evaluating its operation. SUMO was chosen due to the Traffic Control Interface (TraCI) that provides the user to manipulate the behavior of simulated objects in real-time [31]. In SUMO, lanes can be divided into multiple sub-lanes without a limit in the lane resolution while simulated vehicles are also able to interact with other simulated users that are using adjacent sub-lanes [12]. A similar approach can be adapted for the integration of the model in any other microscopic traffic simulation software with a respective API that allows the user to manipulate the simulation extensively such as VISSIM or AIMSUN. After the model was integrated with SUMO, three simulations with a duration of 1800 seconds with 10 simulation steps per second were executed. For each simulation run, all simulated bicycles had the same desired velocity, 10 km/h, 14 km/h and 19 km/h in the first second and third run respectively. The focus of the simulations will be to examine the lateral deviations of the bicycles in space with different velocities and avoid overtaking maneuvers that would lead to greater lateral deviations. The simulated bicycles were restricted to a single bicycle path with a width of 1.6 m. The selected bicycle path width corresponds to the minimum bicycle path width stipulated by the German regulations [32].

Fig. 2 presents the steering angle deviation due to the stabilization process of one randomly selected simulated bicycle for a velocity of 14 km/h and the corresponding lateral position deviation.

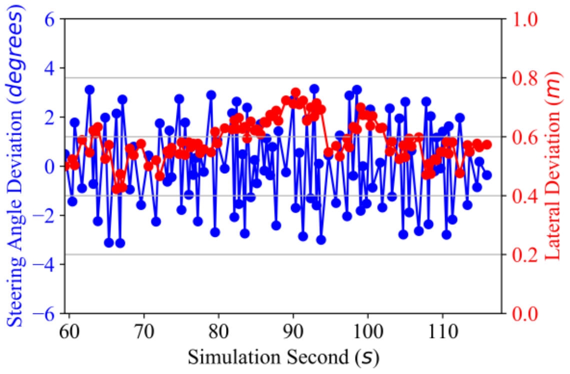

Figure 2. Steering angle deviations and lateral position deviations of a simulated bicycle.

Due to the inclusion of the randomization factor, the amplitude and the frequency of the steering angle deviations vary over time. Although the change of the steering angle may be small, it can sum to a significant lateral deviation from the straight path over time. The maximum lateral deviation for every bicycle is the difference between the maximum and the minimum lateral position of the bicycle in question during the simulation run. Results show that the operational width for a simulated bicycle ranges between 0.46 m for a desired speed of 10 km/h to 0.27 m for a desired speed of 19 km/h. The fact that the average lateral deviation decreases as the bicycle velocity increases was expected due to the exponentially decreasing function that is used to determine the steering angle deviation. Fig. 3 depicts the distribution of the maximum operational width of the simulated bicycles.

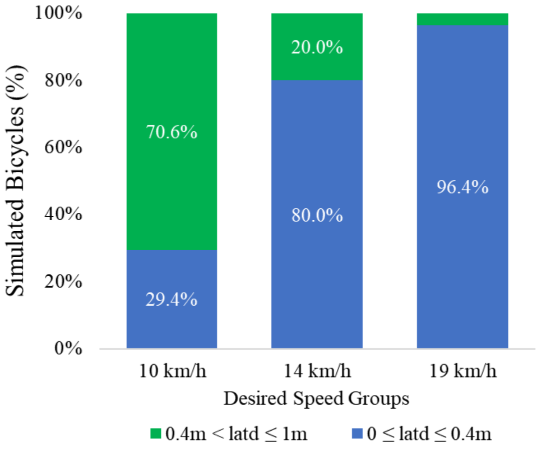

Figure 3. Bicycle operational width distribution.

Results show that 70.6% of bicycles belonging to the desired speed group of 10 km/h require an operational width of at least 0.4 m. The percentage of bicycles that require an operational width of 0.4 m decreases significantly as desired speed increases, since the stabilization process at greater speeds results in smaller steering angle changes. It is expected that under real traffic conditions, where various variable environmental factors affect the stabilization of the person-bicycle system, the resulting average operational widths for all groups may be higher.

Additionally, it is expected that in the case of narrow infrastructure width, bicyclists approaching downstream slower moving bicyclists will not be able to perform an overtaking maneuver due to potential blocking by the slower moving bicycle. This may result in reduced speeds for fast moving cyclists, the creation of platoons or in changing infrastructure in order to perform the overtaking maneuver. Since overtaking bicyclists also must stabilize their own bicycles, it is hard to determine how and when the overtaking maneuver will take place. As a result, of the randomization of the lateral position of the leading bicycle it is difficult to determine if an overtaking maneuver takes place as a part of a tactical decision of the bicyclist or if the bicyclist is able to continue advancing forward due to an acceptable gap that was open for a short time window. The only tactical parts of the proposed model support the overtaking maneuver and limit the bicycle in the present infrastructure if its lateral position changes only due to the proposed model. However, slower moving bicycles may provide space for overtaking if a faster moving bicycle is approaching, especially in the case of limited available width. Thus, the boundary that separates a tactical decision of assisting an overtaking maneuver from the uninfluenced progression in space is unclear and can be the focus of future research.

Nevertheless, the proposed model can be used by a tactical model as well. Besides intensive steering control, lateral deviations from a straight path will be reduced if the bicyclist reduces the pedaling frequency. As the lateral space deviations become smaller, the movement of the bicycle becomes more predictable. This can be integrated into a tactical model as it leads to a reduction of the acceptable overtaking width for a bicyclist, for cooperative overtaking maneuver modelling or for modelling bicycle motion in narrow spaces. Additionally, the operational width value is sensitive to the simulated bicycle speed, when $v \leq 15$ km/h and the randomization parameter of the steering angle. Smaller randomization parameter ranges result in overall lower average operational widths for all desired speed groups. In this study the randomization parameter was conservatively allowed to receive values ranging from 0.001 to 1.1, as no detailed data were available from the field study [26], where a greater range of steering angles is expected due to environmental factors affecting the stabilization process. Still, the use of the randomization factors produces a great variability in observed bicycle trajectories. All simulated bicycles are found to utilize a space of close to 0.8 m for free flow motion. As bicycles can move freely on the full width of the bicycle path, bicycles can be found at any lateral position along the available bicycle lane width at any given time. In a simulation with varying desired velocities, this dynamic behavior will make the execution of overtaking maneuvers more difficult for faster bicycles, which leads to a reduced capacity especially with high bicycle traffic volumes.

V. CONCLUSIONS

In this paper, a model is proposed that introduces aspects of bicycle dynamics in microscopic traffic simulation. The development of the model is based on the review of complex bicycle dynamic models and the experimental results of a study conducted to examine the stabilization process of a human controlled bicycle. As the proposed model is designed for microscopic traffic simulation, simplifications were made in order to preserve the efficiency of the simulation. The proposed model is designed based on laboratory data that do not consider the effects of variable environmental factors on the stabilization process. Appropriate real traffic data should therefore be collected as the steering angle deviations are expected be higher because of the environmental factors. However, despite the conservative approach that was followed in the adaptation of the model in microscopic traffic simulation, the proposed model has already the ability to produce realistic motion trajectories for simulated bicycles. Using the proposed model, simulated bicycles follow waveform trajectories with a varying amplitude of operational width and with varying frequency of steering direction changes. Simulation results showed that individual bicycles with a smaller desired speed require a greater operational width than bicycles with higher speed. This can affect the overtaking behavior of faster moving bicycles, especially in cases of limited infrastructure width. Overall, the proposed model has the potential to improve the accuracy of simulation studies that examine the tactical behavior of bicyclists or the capacity of bicycle infrastructure, especially in collaboration with a fully functional tactical behavior model and when high volumes of bicycle traffic are simulated.

REFERENCES

[1] P. A. Lopez *et al.*, “Microscopic Traffic Simulation using SUMO,” pp. 2575–2582, 2018.

[2] H. Twaddle, T. Schendzielorz, and O. Fakler, “Bicycles in Urban Areas: Review of Existing Methods for Modeling Behavior,” *Transp. Res. Rec. J. Transp. Res. Board*, pp. 140–146, 2014.

[3] L. Huang and J. Wu, “Cyclists’ path planning behavioral model at unsignalized mixed traffic intersections in China,” *IEEE Intell. Transp. Syst. Mag.*, vol. 1, no. 2, pp. 13–19, 2009.

[4] R. Schönauer, M. Stubenschrott, W. Huang, C. Rudloff, and M. Fellendorf, “Modeling concepts for mixed traffic: Steps towards a microscopic simulation tool for shared space zones,” *Transp. Res. Board 91st Annu. Meet.*, vol. 43, pp. 1–16, 2012.

[5] S. Amini, H. Twaddle, and A. Leonhardt, “Modelling of the tactical path selection of bicyclists at signalized intersections,” in *Transportation Research Board 95th Annual Meeting*, 2016.

[6] H. Twaddle and F. Busch, “Binomial and multinomial regression models for predicting the tactical choices of bicyclists at signalised intersections,” *Transp. Res. Part F Traffic Psychol. Behav.*, vol. 60, pp. 47–57, Jan. 2019.

[7] E. Andresen, M. Chraibi, A. Seyfried, and F. Huber, “Basic driving dynamics of cyclists,” in *Simulation of Urban MObility User Conference*, 2013, pp. 18–32.

[8] J. Haifeng, W. Tao, J. Pengpeng, and H. Hun, “Research on cyclists microscopic behaviour models at signalized intersection,” in *16th International Conference Road Safety on Four Continents. Beijing, China (RS4C 2013). 15-17 May 2013.*, 2013.

[9] H. Twaddle and G. Grigoropoulos, “Modeling the speed, acceleration and deceleration of bicyclists for microscopic traffic simulation,” *Transp. Res. Rec. (TRR), J. Transp. Res. Board*, 2016.

[10] H. Twaddle and M. Sc, “Development of tactical and operational behaviour models for bicyclists based on automated video data analysis,” 2017.

[11] H. Twaddle, “Development of tactical and operational behaviour models for byclists based on automated video data analysis,” Technische Universität München, 2017.

[12] M. Semrau and J. Erdmann, “Simulation framework for testing ADAS in Chinese traffic situations,” *SUMO 2016–Traffic, Mobility, Logist.*, vol. 30, pp. 103–115, 2016.

[13] PTV AG, *PTV Vissim 9 user manual*. Karlsruhe: PTV, AG., 2016.

[14] M. Fellendorf and P. Vortisch, “Microscopic traffic flow simulator VISSIM,” in *Fundamentals of traffic simulation*, Springer, 2010, pp. 63–93.

[15] W. Huang, M. Fellendorf, and R. Schönauer, “Social Force based Vehicle Model for Two-Dimensional Spaces,” *Transp. Res. Board 91st Annu. Meet.*, pp. 1–16, 2012.

[16] K. J. Astrom, R. E. Klein, and a. Lennartsson, “Bicycle dynamics and control: adapted bicycles for education and research,” *Control Syst. IEEE*, vol. 25, no. 4, pp. 26–47, 2005.

[17] F. Klein and A. Sommerfeld, *Über die Theorie des Kreisels*, no. 2–3. BG Teubner, 1898.

[18] D. J. N. Limebeer and R. S. Sharp, “Bicycles, motorcycles, and models,” *IEEE Control Syst. Mag.*, vol. 26, no. 5, pp. 34–61, 2006.

[19] J. K. Moore, J. D. G. Kooijman, A. L. Schwab, and M. Hubbard, “Rider motion identification during normal bicycling by means of principal component analysis,” *Multibody Syst. Dyn.*, vol. 25, no. 2, pp. 225–244, 2011.

[20] J. P. Meijaard, J. M. Papadopoulos, A. Ruina, and A. L. Schwab, “Linearized dynamics equations for the balance and steer of a bicycle: a benchmark and review,” *Proc. R. Soc. A Math. Phys. Eng. Sci.*, vol. 463, no. 2084, pp. 1955–1982, 2007.

[21] F. J. Whipple, “The stability of the motion of a bicycle,” *Q. J. Pure Appl. Math.*, vol. 30, no. 120, pp. 312–321, 1899.

[22] M. Dozza and A. Fernandez, “Understanding bicycle dynamics and cyclist behavior from naturalistic field data (November 2012),” *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 1, pp. 376–384, 2014.

[23] J. D. G. Kooijman, A. L. Schwab, and J. P. Meijaard, “Experimental validation of a model of an uncontrolled bicycle,” *Multibody Syst. Dyn.*, vol. 19, no. 1–2, pp. 115–132, 2008.

[24] A. L. Schwab and J. P. Meijaard, “A review on bicycle dynamics and rider control,” *Veh. Syst. Dyn.*, vol. 51, no. 7, pp. 1059–1090, 2013.

[25] J. K. Moore, M. Hubbard, A. L. Schwab, J. D. G. Kooijman, and D. L. Peterson, “Statistics of bicycle rider motion,” *Procedia Eng.*, vol. 2, no. 2, pp. 2937–2942, 2010.

[26] J. D. G. Kooijman, A. L. Schwab, and J. K. Moore, “Some observations on human control of a bicycle,” *ASME 2009 Int. Des. Eng. Tech. Conf. Comput. Inf. Eng. Conf.*, vol. IDETC/CIE, no. March 2016, pp. 1–8, 2009.

[27] J. K. Moore, *Human control of a bicycle*. University of California, Davis Davis, CA, 2012.

[28] J. C. Martin, D. L. Milliken, J. E. Cobb, K. L. McFadden, and A. R. Coggan, “Validation of a mathematical model for road cycling power,” *J. Appl. Biomech.*, vol. 14, no. 3, pp. 276–291, 1998.

[29] D. Allen, N. Rouphail, J. Hummer, and J. Milazzo, “Operational Analysis of Uninterrupted Bicycle Facilities,” *Transp. Res. Rec.*, vol. 1636, no. 1, pp. 29–36, 1998.

[30] V. Cossalter, *Motorcycle Dynamics*. LULU, 2006.

[31] A. Wegener, M. Piorkowski, R. Maxim, H. Hellbrück, S. Fischer, and J.-P. Hubaux, “TraCI : An Interface for Coupling Road Traffic and Network Simulators,” in *Proceedings of the 11th communications and networking simulation symposium*, 2008, pp. 155–163.

[32] FGSV, *Empfehlungen für Radverkehrsanlagen (ERA)*, 2010th ed. Köln: Forschungsgesellschaft für Straßen- und Verkehrswesen (Hrsg.), 2010.